Rational Numbers

Introduction to Rational Numbers

All numbers, including whole numbers, integers, fractions and decimal numbers, can be written in the NumeratorDenominator form.

Rational number

A rational number is a number that can be written in the form p/q, where p and q are integers and q ≠ 0. The denominator of a rational number can never be zero.

e.g. 911, 58, 712

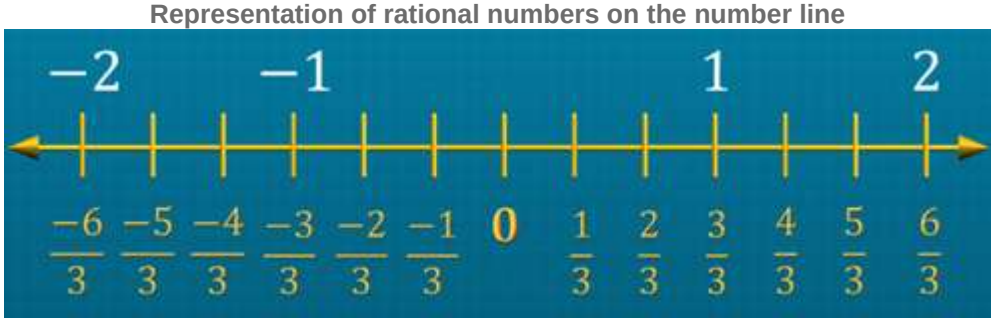
A rational number is positive if its numerator and denominator are both either positive integers or negative integers.

e.g. 25, 34, -7-10, -5-11

If either the numerator or the denominator of a rational number is a negative integer, then it is a negative rational number.

e.g. -25, 3-4, 7-10, 5-11

Representation of rational numbers on the number line



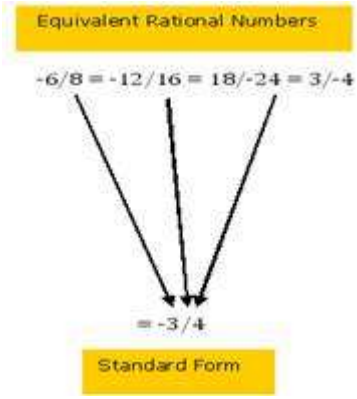
The rational number zero is neither negative nor positive. Positive rational numbers are represented to the right of zero on the number line. Negative rational numbers are represented to the left of zero on the number line.

A rational number obtained by multiplying or dividing both the numerator and the denominator of a rational number by the same non-zero integer, is said to be the equivalent form of the given rational number.

Rational numbers in Standard form

A rational number is said to be in its standard form if its numerator and denominator have no common factor other than 1, and its denominator is a positive integer.

Rational Numbers



To reduce a rational number to its standard form, divide its numerator and denominator by their highest common factor (HCF). To find the standard form of a rational number with a negative integer as the denominator, divide its numerator and denominator by their HCF with a minus sign.

Comparison of Rational Numbers

Among the positive rational numbers with the same denominator, the number with the greatest numerator is the largest. It is easy to compare the rational numbers with same denominators.
e.g. $\frac{28}{30} > \frac{26}{30} > \frac{21}{30}$.

A negative rational number is to the left of zero whereas a positive rational number is to the right of zero on a number line. So, a positive rational number is always greater than a negative rational number.

To compare two negative rational numbers with the same denominator, their numerators are compared ignoring the minus sign. The number with the greatest numerator is the smallest.
e.g. $-\frac{7}{10} < -\frac{3}{10}$; $-\frac{6}{7} < -\frac{4}{7}$

To compare rational numbers with different denominators, they are converted into equivalent rational numbers with the same denominator, which is equal to the LCM of their denominators.

There are unlimited number of rational numbers between two rational numbers. To find a rational number between the given rational numbers, they are converted to rational numbers with same denominators.

Rational Numbers

Operations on Rational Numbers

Addition of Rational numbers

The sum of two rational numbers with the same denominator is a rational number whose numerator is the sum of the numerators of the rational numbers with the same denominator.

To add rational numbers with different denominators, they are converted into equivalent rational numbers with the same denominator.

Additive inverse of a Rational number

Two rational numbers whose sum is zero are called the additive inverses of each other.

e.g. -720 is the additive inverse of 720 and 720 is the additive inverse of -720.

Subtraction of Rational numbers

The difference between two rational numbers with the same denominator is a rational number whose numerator is the difference of the numerators of the rational numbers with the same denominator.

To subtract rational numbers with different denominators, they are converted into equivalent rational numbers with the same denominator.

Multiplication of Rational numbers

The numerator and denominator of the product of two rational numbers are equal to the product of their individual numerators and denominators.

The numerator of the product of a rational number and an integer is equal to the product of the numerator and the integer with the same denominator.

Reciprocal of a Rational number

Two rational numbers whose product is 1 are called reciprocals of each other. A rational number and its reciprocal will always have the same sign.

e.g. $-136 \times 6-13 = 1$

Division of Rational numbers

To divide one rational number by another, first number is multiplied with the reciprocal of the second number.

e.g. $518 \div 29 = 518 \times 92 = 54$